

## EXAMINING OPTIMAL CONTROL THROUGH THE APPLICATION OF (LQR) AND (LQT) APPROACHES IN THE CONTEXT OF DC MOTORS.

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### ABSTRAK

Berbagai sektor industri memiliki kebutuhan untuk motor DC karena kelebihanannya, seperti torsi yang substansial, tidak ada rugi daya reaktif, tidak ada gangguan pasokan listrik, dan kontrol yang presisi. Otomasi dalam pengendalian memainkan peran penting dalam kemajuan ilmu pengetahuan dan teknologi. Sistem kontrol yang efektif harus memenuhi kriteria tertentu, mencakup indeks kinerja yang berkaitan dengan akurasi, stabilitas, dan respon yang cepat. Sistem kontrol optimal adalah salah satu yang dibentuk melalui optimasi indeks kinerja, dianggap optimal ketika nilai parameter dipilih untuk memaksimalkan atau meminimalkan indeks kinerja yang dipilih. Dalam konteks motor DC. Dalam konteks pengaturan kecepatan menggunakan teknik kontrol optimal Linear Quadratic Regulator, optimasi indeks kinerja melibatkan konfigurasi nilai matriks  $Q$ . Ini, pada gilirannya, menghasilkan matriks penguat umpan balik  $K$  dan matriks pelacakan optimal  $L$  untuk motor DC. Proses ini diulang untuk pengaturan kecepatan pada motor DC menggunakan teknik kontrol optimal Linear Quadratic Regulator, di mana optimasi indeks kinerja dicapai dengan menyesuaikan nilai matriks  $Q$ , yang mengarah pada derivasi matriks penguat umpan balik  $K$  dan matriks pelacakan optimal  $L$ .

**Kata Kunci:** Pemodelan motor DC, optimasi, dan penerapan teknik Linear Quadratic Regulator dan Linear Quadratic Tracking.

### ABSTRACT

Different sectors of industry have a need for DC motors due to their benefits, such as substantial torque, absence of reactive power loss, no disruption to power supply, and precise control. Automation in control plays a crucial role in advancing science and technology. An effective control system must fulfill specified criteria, encompassing performance indices associated with accuracy, stability, and rapid response. An optimal control system is one fashioned through the optimization of performance indices, considered optimal when parameter values are chosen to maximize or minimize the selected performance index. In the context of DC motors. In the context of speed regulation using the Linear Quadratic Regulator optimal control technique, the optimization of performance indices involves configuring the  $Q$  matrix value. This, in turn, results in the generation of the  $K$  feedback amplifier matrix and the optimal  $L$  tracking matrix for the DC motor. The process is repeated for speed regulation in the DC motor using the Linear Quadratic Regulator optimal control technique, where performance index optimization is achieved by adjusting the  $Q$  matrix value, leading to the derivation of the  $K$  feedback amplifier matrix and the optimal  $L$  tracking matrix.

**Keyword :** DC motor modelling, optimization, and the application of Linear Quadratic Regulator and Linear Quadratic Tracking techniques.

## 1. INTRODUCTION

### 1.1 Background

DC motors, or direct current motors, stand as one of the extensively employed electric motor types in

various industries [1]. Their enduring usage is attributed to their commendable regulatory traits. Examining a DC motor system involves the mathematical modeling of pertinent variables [2].

This mathematical model, denoted as a transfer function, is integrated into the fcn transfer block and simulated using Matlab Simulink [3]. The transfer function commonly employed is an equation of a specific order.

1. A DC motor, or direct current motor, transforms electrical energy from DC into mechanical energy, specifically rotation [4]. The structural configuration of a DC motor closely resembles that of a DC generator, comprising three primary components:

- Stator-mounted field coils
- Rotor-based anchor coil
- An air gap separating the field coil and armature coil.

2. A DC motor, also known as a direct current motor, performs the conversion of DC electrical energy into mechanical energy, manifesting as rotational motion [5]. Its physical structure closely mirrors that of a DC generator, consisting primarily of three components:

- Field coils situated on the stator
- An anchor coil positioned on the rotor
- A gap of air between the field coil and armature coil.

**C23 Typical Outline Drawing**

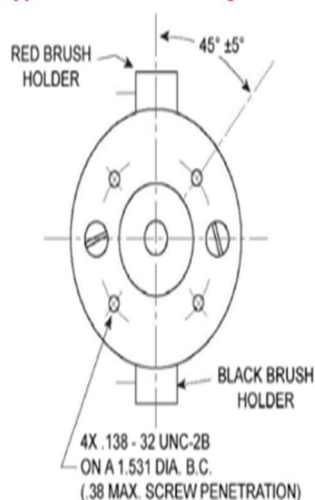


Figure 1. C23 Typical outline drawing

A control system is a mechanism that achieves the intended response by regulating the output [6]. Crafting a precise control system model involves simulation, offering a more convenient and cost-effective alternative to direct operation on an actual system [7]. Through this simulation, the system's behavior and attributes are discerned, facilitating the evaluation of the designed system's performance [8]. Consequently, following the simulation, a refined control system is derived, prepared for implementation in real-world applications. Automation has become a crucial component in the advancement of science and technology [9]. The

progress in the field of automation offers several advantages, including the elimination of tedious human tasks and the potential for increased production capacity [10]. Optimal control, in particular, enhances system performance and precision [11]. Addressing the optimal control system challenge requires establishing guidelines for optimal control decisions aimed at minimizing deviations from ideal conditions [12]. Typically, this criterion is rooted in the system's performance index. An effective control system is characterized by both rapid and stable responsiveness without excessive energy consumption [13]. Achieving such a system involves defining the appropriate performance index, ultimately resulting in an optimal control system. This study employs a DC motor as the plant system, intending to apply control through the Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking techniques [14]. The optimization of the performance index involves fine-tuning the Q matrix value, leading to the generation of a feedback amplifier matrix (K) and an optimal tracking matrix (L) for the DC motor's performance index [15]. To facilitate control through the LQR and LQT methods, the DC motor to be regulated must first determine its transfer function, enabling its conversion into a state space for inclusion in the calculation process [16].

## 1.2 Phase of the study

### 1.2.1 Method for recognition

In the initial phase of this investigation, the process commences with the identification of the DC motor model slated for testing, coupled with an examination of LQR and LQT methodologies. This involves:

- Selecting appropriate values for the weight matrix Q and R.
- Calculating the feedback matrix K.
- Simulating the closed-loop response of the system.

### 1.2.2 Attributes of the system

System characteristics refer to distinctive features or specifications defining the performance of a system. The system's output response emerges following the application of an input or test signal [17]. System responses are categorized into time response characteristics and frequency response characteristics [18]. This study focuses on examining the time response characteristics of the DC motor, aiming to observe how the system output responds to changes over time [19].

### 1.2.3 Mathematical representation

A basic portrayal of a problem or occurrence conveyed through mathematical concepts [20]. Subsequently, a problem is formulated based on the observed issues [1]. The presence of this

mathematical model simplifies the problem-solving process. LQR Technique. Expense Functions :

$$J = \int_0^{\infty} (x^t Q x + u^t R u) dt$$

Q represents the state weighting factor (a positive semi-definite matrix), while R signifies the weight of the control variable factor (a positive definite matrix) [21]. The control signal is denoted as "u," and the optimal value of K is determined for the performance index [22].

$$K = R^{-1} B^T P$$

In the provided equation, the matrix P must meet the conditions outlined in the reduced equation [23].

$$A^T P + P A - P B R^{-1} B^T P + Q = 0$$

Seeking a positive definite matrix for the P value, substitute this P value into the equation mentioned earlier to obtain the optimal K value [24].

Utilizing the Ziegler-Nichols Tuning method to determine the transfer function of a DC motor [25].

Conversion through Laplace transform:

$$\tau s T_s + T_s = T_i s$$

$$\frac{T(s)}{T_i(s)} = \frac{1}{\tau s + 1}$$

Electric properties:

$$\frac{di_a}{dt} = \frac{R}{L} i_a - \frac{K}{L} \omega_r + \frac{1}{L} V_a$$

$$\frac{d\omega_a}{dt} = \frac{K}{J} i_a - \frac{B}{J} \omega_r$$

Physical attributes:

$$T = K i_a$$

T represents the torque generated by the motor.

State space model:

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega_a}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{di_a}{dt} & -\frac{K}{L} \cdot \frac{K}{J} - \frac{B}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_a$$

$$= Ax + Bu$$

$$Y = [0 \ 1] \begin{bmatrix} i_a \\ \omega_r \end{bmatrix} + [0] V_a$$

$$= Cx + Du$$

$$u = -Kx$$

#### 1.2.4 Analysis of characteristics with a first-order nature

A system is considered first-order when it involves a variable "s" with the highest power being one [1]. The mathematical representation of a first-order system for a DC motor can be expressed as follows:

$$G(s) = \frac{K}{T + sK}$$

$$K = \frac{T}{i}$$

Data: T stands for torque (Nm), I represent current (A).

### 1.3 Fundamental Principles

Electric motors running on direct current DC motors, or direct current motors, are among the extensively employed electric motors in various industries [1]. These machines are expected to persist in usage due to their favorable regulatory characteristics. The fundamental principle revolves around the interaction between two magnetic fluxes, namely the field coil and the armature coil. The energy generated is manifested in the form of rotational motion.

Linear Quadratic Regulator (LQR) and Linear Quadratic Tracking (LQT) refer to control techniques. Specifically, Linear Quadratic Regulator (LQR) is a control method. (2.2.2) It is a contemporary approach to designing constrained systems based on state space. The LQR controller, operating within this framework, involves two crucial parameters: the weight matrices Q and R. These matrices must be appropriately determined to generate optimal control actions. LQR achieves control over the process or plant by utilizing a linear combination of plant states. Linear Quadratic Tracking, on the other hand, is an optimal control technique designed for linear systems with quadratic criteria, specifically addressing tracking problems. The initial stage in designing an LQR controller involves selecting appropriate values for the weight matrices Q and R. It is essential to assign a higher weight to the input R compared to the state, with the state weight exceeding that of the input. Subsequently, the feedback matrix K can be computed, and the closed-loop response of the system can be determined through simulation. In LQR and LQT controller design, the choice of Q and R weight matrices is guided by the principle that a higher value of Q brings the system closer to the minimum point, and a greater value of R results in reduced energy consumption.

Software known as Matlab Matlab is a programming platform tailored for numerical processing. Developed by MathWorks, it employs a matrix-based language, enabling tasks such as data analysis, algorithm creation, and model and application development. Matlab incorporates Simulink, a feature serving as a graphical programming tool for dynamic system simulations. The simulation process utilizes a functional diagram comprising interconnected blocks representing their respective functions.

### 1.4 Modeling a DC Motor

Standard representation of a first-order transfer function

$$G(s) = \frac{Ks}{\tau s + Ks}$$

$$Ks = \frac{T_s}{i}$$

$$Ks = \frac{1}{5,1}$$

$$Ks = 0.372 \text{ Nm/A}$$

$$G(s) = \frac{0,0372}{0,19s+0,0372}$$

Details:


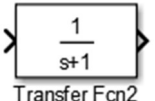
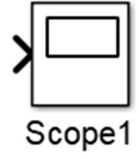
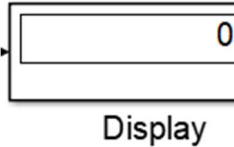

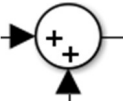


K represents the coefficient,  
τ stands for DC motor torque,  
I denote DC motor current.

Thus, the first-order equation for a DC motor is derived:

$$G(s) = \frac{Ks}{T+K}$$

$$G(s) = \frac{0,0372}{0,19s+0,0372}$$

Table 1. Specification for Simulink MATLAB components

Catalog of Simulink elements		
Element	setting	methods of operation
 Step	Linked to addition and presentation	In the context of system input
 Transfer Fcn2	Linked to summation, step, scope, and presentation	Handle input based on the transfer function equation
 Scope1	At the output point of the bus creator	presents a graphical representation of the system outcomes
 Display	At the output of the fcn transfer	shows the outcomes of a system
 Step	Linked to addition and presentation	In the context of system input
		 Gain

### 1.5 Diagram of the System Blocks

It forms an element within a simulation diagram, illustrating each component, section, or operation of the process through interconnected blocks. These blocks are linked by lines, providing a visual representation for the simulation. The block diagram encompasses inputs, processes, and outputs integral to the ongoing simulation.

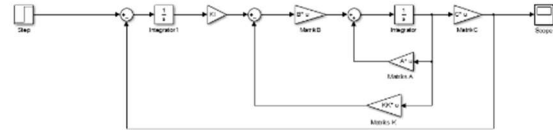


Figure 2. Diagram of the system blocks

### 1.6 System Flowchart

Flowcharts serve the purpose of illustrating the stages or procedures to be simulated in a program. Typically, process flowcharts are presented as diagrams that encompass input, processes, and culminate in output. The visual representation of the flowchart is available in the accompanying image.

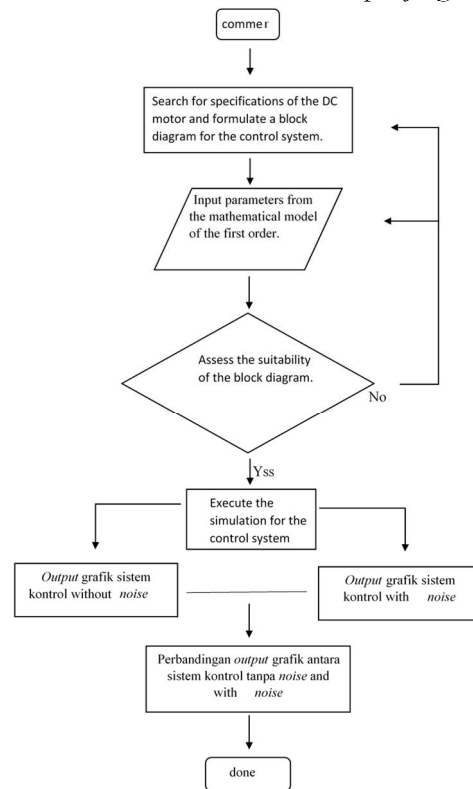


Figure 3. System flowchart

## 2. RESULT AND DISCUSSIONS

### 2.1 Specifications of the DC Motor

Table 2. Specifications for DC motor model C23-L50

Performance data	
Rated voltage (V)	12 VDC
velocity	1600 RPM
mean torque	0.19
Reduction rate	22
Nominal current (A)	5.1 Amps

Characteristics of DC Motor Model C23-L50		
Parameter	Symbol	Besar dan satuan
Rotor inersia	$J_m$	0.0000459 Kg.m <sup>2</sup>
Damping	$B$	0.001 N.m/(rad/s)
Rated Torque	$K_t$	0.19 N.m/A
Back EMF	$K_E$	0.0519 V/(rad/s)
Refusal	$R_T$	0.63 Ohm
Inductance	$L$	0.00077 H

### 2.2 A Sequence of Trials

Linear Quadratic Regulator circuit in a noise-free environment.

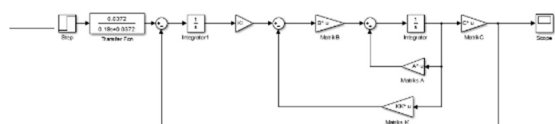


Figure 4. LQR circuit in a noise-free environment

Figure 4 displays the LQR circuit featuring a first-order FCN transfer obtained through the aforementioned calculation. The outcomes depicted on the scope are observable in Figure 5.

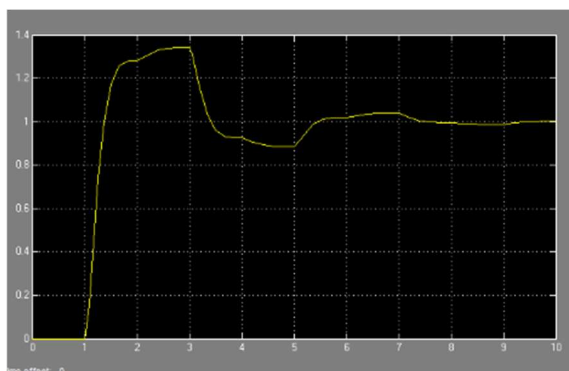


Figure 5. The outcomes depicted on the scope

The results in Figure 5 show that the DC motor used to test the optimal system with LQR is still not stable to order 1, as shown by the results of the overshoot that occurs and the stability value that does not occur immediately within a few seconds.

With noise

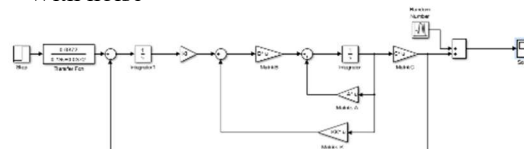


Figure 6. LQR circuit with noise

Figure 6 illustrates an LQR circuit that includes introduced noise for assessment.

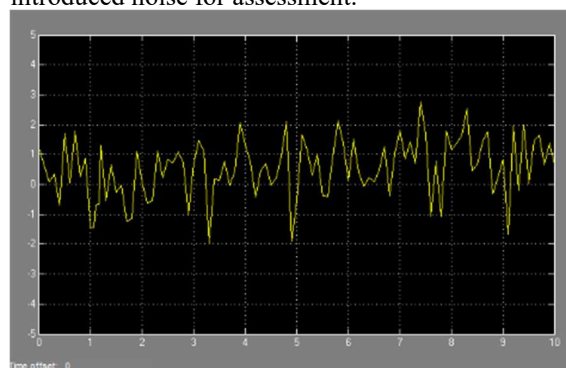


Figure 7. Noise in the LQR circuit leads to variations in the system waves

The presence of noise in the LQR circuit leads to variations in the system's waves, preventing them from maintaining a constant position.

## 3. CONCLUSION

From the outcomes of the simulation tests conducted in Matlab, one can infer that the effectiveness of the LQR controller with a first-order is insufficient to optimize the DC motor system. The circuit data indicates a lack of stability within a brief timeframe, suggesting that the current setup is far from achieving optimal performance for the DC motor.

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